# Session of History of Mathematics

## *Transfer and Scientific Exchange Relations between German and Italian Mathematicians, from the 17th to the mid-20th Century*

# A b s t r a c t s

## An Unpublished Memoir by Lagrange in Berlin

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The memoir on life annuities was written by Lagrange at the beginning of 1776 and presented to the Académie des Sciences of Berlin on the 22nd of February. It contains a critical analysis of the rules of an insurance fund for widows which was to guarantee a pension based on an initial capital paid by the husband, followed by an annual premium during his life. Similar institutions were being founded in this period in various regions of Germany (Hannover, Berlin, and Gotha), as well as in Denmark. The initial project of the institute, put forward by Augustin Kritter in 1768, had been examined by Euler, from the mathematical point of view, in a memoir which was published in German in Hamburg in 1770, and later in an amplified version in French in 1776 in St. Petersburg, within his most famous work on actuarial mathematics. In this work Euler solved the problem: "In the case of procuring for person *B* aged *b* a life annuity of 100 roubles per year, which may not commence until after the death of another person *A*, aged *a*, the question is at what price the present expectation of the aforementioned person *B* is to be estimated, in order to achieve the aforementioned pension". The problem that Lagrange dealt with was different, because when the Prussian institute was established, seven years after Kritter's proposal, the rules had been changed: in particular, in the case of the wife's premature death or divorce the initial capital was to be given back to the heirs. Lagrange's memoir, which is now being published, presents, in 82 folios, a detailed analysis of the problem and foresees bankruptcy for the Berlin institute. The political reaction and polemics which the memoir aroused prevented its publication. The Prussian Institute for the Subsistence of Widows came under a series of critical articles on the part of Kritter, but it carried on its work until at least 1793.

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# German Mathematics and Absolute Differential Calculus

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The purpose of this lecture is to analyze the various aspects of the influence of German mathematics on the genesis of absolute differential calculus.

Emerging at the end of the 19th century with the work of the Italian mathematician G. Ricci-Curbastro, this theory may primarily be viewed as a variation of a central topic of German mathematics, i.e. the study of the invariants of Riemannian geometry. This topic was extensively developed after the publication of Riemann's *Habilitationsschrift*, in particular by E.B. Christoffel and R. Lipschitz. Most of the concepts of tensor analysis – such as the notions of tensor and covariant differentiation – were already present in this embryonic stage of development of the theory from an 'algorithmical' point of view.

However, despite the fundamental significance of this tradition to Ricci-Curbastro's work, it may be held that the real genesis of such concepts finds its true justification only when one takes into account a second tradition of research on differential invariants; that is, the tradition that was concerned with the investigation of 'differential parameters', as arising out of the work of the French mathematician G. Lamé and developed mainly through the research work of E. Beltrami.

In spite of its original character, this second tradition of research tends to display the presence of other forms of influence of German mathematics on the genesis of absolute differential calculus. This essentially concerns Ricci-Curbastro's consideration of the mathematical idea of invariance – conceived as a form of convergence of different (algebraic, geometric, differential) contexts of research – above all in connection with Klein's *Erlangen programme*.

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# The influence of Grassmann on the School of Peano

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In this talk we present a historical reconstruction and analysis of the theoretical development which, in the context of the School of Peano, led from H. Grassmann's legacy to the realization of vector calculus and the theory of homographies.

It should be remembered that Hermann Günther Grassmann published his *Ausdehnungslehre* in 1844, a work full of philosophical reflections, written in language that had little to do with the mathematical mentality. In Germany this work made no impression on the mathematical world; but in Italy Giusto Bellavitis read it and began an exchange of letters with Grassmann. It was also appreciated by Luigi Cremona. In 1862 Grassmann published a second edition in which he dedicated considerable space to geometrical interpretations and applications, but it was no more successful than the first.

When from the academic year 1885-86 to that of 1888- 89 Giuseppe Peano (1858-1932) held the post of lecturer in "Geometrical applications of infinitesimal calculus" at the University of Turin, he was well aware of the problems regarding geometric calculus, so that, when in 1887 he published his lectures in a book entitled *Applicazioni geometriche del calcolo infinitesimale*, the authors he had in mind were Bellavitis, Möbius, Hamilton and Grassmann. In particular, in this first treatise on the subject, he gave importance to Bellavitis' manner of expression, in part because of the influence of his colleague Angelo Genocchi, who was linked to Bellavitis by friendship and respect. But it was in 1888 that Peano published the basic work on these topics: *Calcolo geometrico secondo l'*Ausdehnungslehre *di H. Grassmann preceduto dalle operazioni della logica deduttiva*, a work crucial also for the history of logic. Here he shows that he is decidedly convinced by Grassmann's approach, making reference to the 1844 edition of the *Ausdehnungslehre*.

In the School of Peano who devoted himself above all to the studies of geometric calculus was Cesare Burali Forti (1861-1931); but Filiberto Castellano (1860-1919), Tommaso Boggio (1877-1963) and Mario Pieri (1860-1904) also took an interest in the subject. According to Peano and to Burali-Forti, the co-ordinates method constitutes a numerical intermediation for the study of geometrical objects and their properties. Geometric calculus, with its absoluteness and conciseness, is an immediate, direct approach to geometrical problems, though without excluding co-ordinates. The studies of Burali-Forti

and Marcolongo develop along the same lines as those of Grassmann and Peano, as regards vectorial calculus.

In this lecture our aim is also to attempt a generalization of fundamental ideas introduced by Peano (and by H. Grassmann). In addition we will analyze the applications of Peano's geometric calculus to demonstrations of some fundamental projective geometrical theorems, and, finally, we will examine an important physical mathematical application of the homographies of Roberto Marcolongo on Lorentz transformations.

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# The Reception of Pietro Mengoli's Work on Series by Leibniz (1672-1676)


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In September 1672 Christiaan Huygens tested the mathematical abilities of the young Leibniz by proposing to him that he find the sum of the series of reciprocal triangular numbers. After presenting his successful invention to the Royal Society of London in 1673, Leibniz was told that the result had already been published by Pietro Mengoli in 1650.

The subsequent correspondence with Henry Oldenburg offered Leibniz more information about Mengoli's work on series. During the same period, Mengoli was engaged in a controversy about number theoretical problems proposed by Leibniz's acquaintance

Jacques Ozanam. In April 1676 Leibniz had the opportunity to study Mengoli's *Circolo* (1672) in detail, as is documented by the extracts surviving in his manuscripts.

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## Karl Weierstraß's Correspondence with Italian Mathematicians


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It was a well-known fact that Weierstraß himself had not kept parts of his correspondence and had even destroyed some of it. It was therefore an enormous and very welcome surprise when I succeeded in detecting a considerable part of his *Nachlass* comprising a large number of letters sent to Weierstraß (see SCHUBRING 1998).

The majority of these letters were written by German colleagues; in particular Kronecker, Koenigsberger, and Du Bois-Reymond stand out. Among the foreigners, the best represented are French and Italian mathematicians.

As a part of my ongoing project to prepare the edition of this extant Weierstraß-correspondence, I will present the letters by the Italian mathematicians and speak about the extant complementary letters by Weierstraß to his correspondents in Italy. The earliest and at the same time the most constant correspondent was Casorati; among the others were Beltrami, Cremona, Genocchi, Pincherle, C. Segre. Not all letters written to Weierstraß are extant, however.

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